Static and Free Vibration Analysis of a Sandwich Beam with Partial Constraining Layer with MR Fluid Core

A Thesis Submitted to

National Institute of Technology, Rourkela

In partial fulfilment of the requirement for the degree of

Master of Technology

in

Mechanical Engineering

(Specialization-Machine Design and Analysis)

By

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National Institute of Technology

Rourkela-769008 (India)

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Under the guidance of

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Rourkela-769008 (Orissa), INDIA

CERTIFICATE

This is to certify that the thesis entitled "**Static and Free Vibration Analysis of a Sandwich Beam with Partial Constraining Layer with MR Fluid Core**" submitted to the National Institute of Technology, Rourkela by Nihar Saikia Roll No. 213ME1390 for the award of the Master of Technology in Mechanical Engineering with specialization in Machine Design and Analysis is a record of bonafide research work carried out by him under my supervision and guidance. The results presented in this thesis has not been, to the best of my knowledge, submitted to any other University or Institute for the award of any degree or diploma.

The thesis, in my opinion, has reached the standards fulfilling the requirement for the award of Master of Technology in accordance with regulations of the Institute.

Place: Rourkela

Date: 28th May, 2015

(Prof. S. C. Mohanty)

Department of Mechanical Engineering

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Date: - 28th May 2015

Place: - Rourkela

(Nihar Saikia)

Contents

ABSTRACT

The present work deals with the static and free vibration analysis of a beam with partially covering constraining layer with magnetorheological fluid core. For the analysis fixed free end cantilever has been considered. The beam has been modelled using finite element method. The sandwich beam element has six degrees of freedom per node, whereas the base beam element has three degrees of freedom per node. The governing equation of motion have been derived using Hamilton's principle in conjunction with finite element method. The effect of magnetic field strength and constraining layer position on the frequency and critical buckling load of the beam has been studied. It has been found that increase in magnetic field strength enhances the first three mode frequencies. Constraining layer nearer to the free end gives highest value of frequencies. Maximum buckling capacity is achieved by placing the constraining layer at the mid position of the beam. Experiments have been carried out to validate a few theoretical findings.

Keywords: Magnetorheological fluid core, FEM, Magnetic Field

List of figures and Tables

NOMENCLATURE:

$\{\Delta^{(e)}\}$: Nodal Displacement

u_i : Axial Displacement

w_i : Transverse Displacement

θ_i : Rotary Moment

U : Potential Energy

T : Kinetic Energy

K : Stiffness Matrix

M : Mass Matrix

G_v : Complex Shear Modulus

γ_v : Shear Deformation

ϵ_{vl} : Longitudinal Deformation

ϵ_{vT} : Transverse Deformation

G_v : Complex Shear Modulus

G_v^* : In Phase Shear Modulus

G_r : Shear Modulus of rubber

K_g : Element Geometric Matrix

P : Axial Load

L : Length of the specimen

G : Magnetic Field Strength Unit i.e. Gauss

1.1INTRODUCTION

Damping is very important in substructures and systems which are being subjected to either force of dynamic nature or shock. One of the many alternative ways to control the noise which is being generated and the vibration been produced in structures and systems is the passive damping treatment. The traditional or conventional passive control methods include use of absorbers, barriers, dampers, silencers, etc. which are subjected to excitation of dynamic origin or shock in nature. For systems of frequency of excitation of external origin, either modifying in system stiffness value or mass value reduces the unwanted vibrations as the above mentioned parameters have an effect on the resonant frequency of so called system. But in most of the real cases, the dampening of vibrations may use isolators or damping materials for the reduction in vibration. Advancement in technology which are used in the fabrication of materials and more usage of sophisticated analytical and modelling techniques to study the dynamic characteristics of materials and structures facilitates the user to improve the reduction in vibration of a system. Viscoelastic materials shares both the characteristics and properties of viscous fluid and elastic solid materials. There are mainly two methods of usage of viscoelastic material. First one in the constraining layer and the other one in free layer treatment or the unconstrained layer. Depending on the usage sandwich structure utilizes the constrained layer treatment method in obtaining better properties of all the layers in the sandwich beam. In the above mentioned case the viscoelastic material is sandwiched between the structure surface and the layer of metallic material. The conventional sandwich construction includes a relatively thick core of a material of low density, being sandwiched between the top and bottom face sheets which are the face layers of relatively thinner size.

1.2 DAMPING

Damping refers to the extraction of mechanical energy from a system in vibration usually from conversion of the energy to heat. Damping serves to control the steady-state resonant response. There are two types of damping: material damping and system damping. Material damping is the damping inherent to the material while system damping includes the damping at the supports, boundaries, joints, interfaces, etc. Various terms such as viscous damping, hysteretic damping, Coulomb damping, linear and proportional damping, etc. represent vibration damping. The following types of damping are normally seen:

1. Coulomb /dry friction damping
2. Material / solid damping
3. Viscous damping
4. Viscoelastic damping

COULOMB / DRY FRICTION DAMPING

The damping force is constant is magnitude but opposite in direction to the motion of the vibrating body. It is due friction between rubbing surfaces which are either dry or with insufficient lubrication.

MATERIAL / SOLID DAMPING

As the materials are deformed, energy is being either absorbed or dissipated by the material. This effect is due to friction between the internal planes, which either slip or slide with the deformation. When a body with material damping is being subjected to vibration, the stress-strain diagram shows a hysteresis loop. The area of this loop denotes the energy lost per unit volume of the body per cycle due to damping.

VISCOUS DAMPING

Viscous damping is the most commonly used damping mechanism in vibration analysis. When a mechanical system vibrates in a fluid medium such as air, gas, water, and oil, the resistance offered by the fluid to the body causes energy to be dissipated. In this present case, the amount of dissipated energy depends on factors such as the size and shape of the vibrating body, the viscosity of the fluid, the frequency of vibration, and the velocity of the vibrating body. In viscous damping, the damping force is proportional to the velocity of the vibrating body.

VISCOELASTIC DAMPING

Viscoelasticity may be defined as material response that exhibits characteristics of both a viscous fluid and an elastic solid. A viscoelastic material (VEM) combines the two properties to return to its original shape after being loaded. The degree to which a material behaves either viscously or elastically depends mainly on temperature and rate of loading (frequency). Many polymeric materials having long chain molecules exhibit viscoelastic behaviour such as plastics, rubbers, acrylics, silicones, vinyl's, adhesives, urethanes, epoxies, etc. The use of complex modulus brings a lot of convenience in studying the material properties of viscoelastic materials. The material properties of viscoelastic materials depend significantly on environmental conditions such as environmental temperature, vibration frequency, pre-load, dynamic load, environmental humidity and so on. Therefore, a good understanding of such effects, both separately and collectively, on the variation of the damping properties is necessary in order to tailor these materials for specific applications.

1.3 FINITE ELEMNT METHOD

In finite element, initially the domain is subdivided into many sub domains or elements which are called finite elements. The computation of the solution or its approximation is been done on selected points called nodes. The approximate solution in the given node gives rise to the approximate solution of the geometrical figure. The approximate solution becomes exact in two cases: One dividing the domain into infinite number of sub domain. And the other one primary variable should contain a whole set of polynomials.

Steps involved in Finite Element method:

1. Discretize and select the element types.
2. Select a primary variable function.
3. Defining Relations.
4. Deriving the element stiffness matrix and equation.
5. Assemble the matrix and substitute the boundary equation.
6. Solve for primary and secondary unknowns.
7. Interpret the results.

Applications of finite element method:

Structural Areas:

1. Stress Analysis.
2. Buckling Analysis.
3. Vibration Analysis.

Non Structural Areas:

1. Heat Transfer.
2. Fluid Mechanics.
3. Distribution of electric and magnetic potential.

Advantages of Finite Element method:

The following advantages are shown below:

1. Easy modelling of complicated shapes.
2. Handling of general load conditions without difficulty.
3. Handling of any number of boundary conditions.
4. Includes dynamic effect.
5. Handling of nonlinear behaviour with large deformations.

Limitations of Finite Element method:

1. The value of stress accuracy depends on mesh size.
2. The approximate solution may not be accurate.

1.4 SANDWICH STRUCTURE:

A sandwich-structured composite is a special class of composite materials which is fabricated by attaching two thin layers but with high stiffness compared to a lightweight. The core material is normally a low strength material, but its higher thickness provides the sandwich composite with high bending stiffness with overall low density. The strength of the composite material is dependent largely on two factors the outer skins and the interface between the core and the skin.

1.4.1 MATERIAL PROPERTIES OF THE SANDWICH STRUCTURE:

Depending on the function performed in which the sandwich beam is utilized, the material properties are being varied. The advancement in newer technology in composite materials and functionally graded materials had open many options for the researchers in choice of material selection for varied purposes. The choice of material in sandwich structure depend mainly upon the property or the desired need of purpose for example high strength, high temperature resistance, surface finish to name a few. The number of cores which are been available in recent times has increased too many folds due to the introduction of more advancement in cellular plastics. The combining options of the face sheet materials with core materials of varied characteristics make us used to play a role in a wide range of applications which are of immense importance.

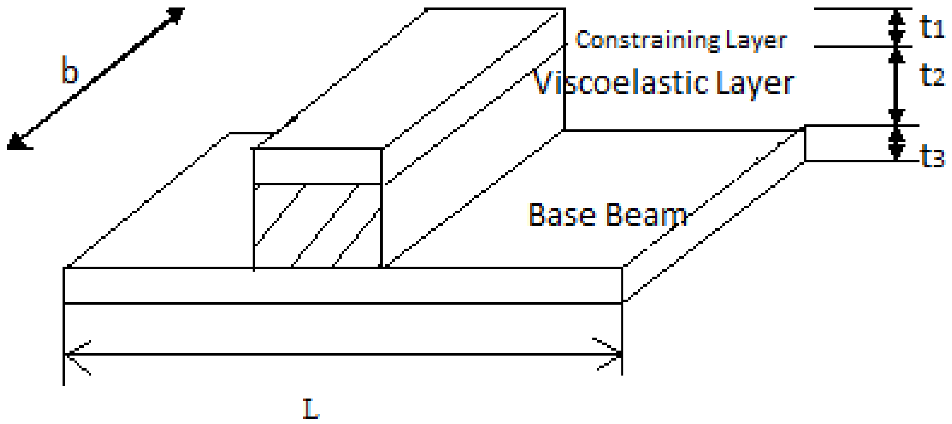


Fig 1: Sandwich Model

The designer should have adequate information on the properties of the material such as stiffness for the better analysis and usage of the material used in the sandwich structure. Obtaining results on the various properties dealing with the sandwich structure one should do experimental work as it is very hard to obtain the results theoretically. In sandwich structures one would find difficult in maintaining a relation between the varied properties but it can be a great boon if one uses it effectively. The properties which is fruitful to one's result may not show better results in some other cases, so it depends totally on individual choices to use which properties. The primary objective of any researcher or engineer would be to obtain better results from the existing properties which would be helpful for the future generation.

The materials used in the core are used to support the beam as well as not to deform under the application of load. They serve the function of holding the beam to a particular position and do not fail under the applied force. The core material should be of low density material, dampening the vibration and noise, of high shear modulus, higher stiffness normal to the face sheet and in addition of the above properties should be thermally insulated. The following materials are used as the core material polymeric foam, polyethylene terephthalate, polymethacrylimide, PVC (polyvinylchloride), wood cores, and honeycomb cores.

The top and the bottom layers of the sandwich structure are the base layer and the constraining layer. They are in the form of sheets. The properties in the layers which are ask for are the high impact, wear, chemical and heat resistance, better tensile and compressive properties along with high stiffness which give rise to high flexural rigidity. The layers should have high degree of surface finish. The materials which are being used for face materials are the metal and alloys, composites, wood etc.

1.4.2 DESIGN CONSIDERATIONS OF THE SANDWICH STRUCTURE

The sandwich structure prepared are design to satisfy the following criteria:

- The face sheets should be able to sustain tensile, compressive as well as shear stress.
- The core sheet should be able to withstand the shear stress as well as overall buckling of the beam.
- The core sheet should have sufficient compressive strength.
- The sandwich beam must have necessary flexural rigidity as well as shear rigidity to sustain deflections on the application of load.
- The sandwich beam should hold the structural virtue on account of application of force.

1.4.3 APPLICATION AREA OF THE SANDWICH STRUCTURE

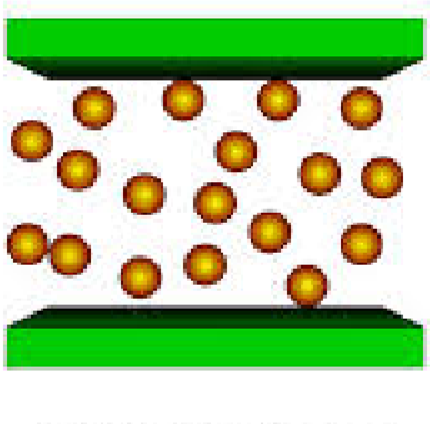
The sandwich structures has varied applications and can be used in the following areas:

- Aerospace industry

- Construction Industry
- Ship Building
- Railway Manufacturing
- Automobile Industry

1.5 SANDWICH STRUCTURES WITH MAGNETORHEOLOGICAL ELASTOMERS

The sandwich specimen along with the use magnetorheological elastomers (MRE’s) can drastically improve the properties of the sandwich beam. Magnetorheological fluids are special class of fluids which on subjected to magnetic field drastically increases the viscosity of the fluid to the level of being a viscoelastic material. The properties of the MRE’s such as the shear modulus, damping factor can be improved by the use of magnetic field. Therefore they can be used as an effective dampener to the vibration on the application of the load. Silica oil with definite proportion of iron particles can be effectively used as MRE’s. On the application of magnetic field the iron particles get aligned in the same pattern, which eventually lead the MRE’s to behave as viscoelastic material which effectively reduces the dampening effect. The core material is normally poured of MRE’s as it is the most risky part due to the high shear stress acting on it. Thus, MRE’s can be used in the core material, which absorbs the vibration due to the application of load and can dampen the vibration induced. Commonly referred MRE’s fluid are oleic acid, citric acid, tetramethylammonium hydroxide etc.



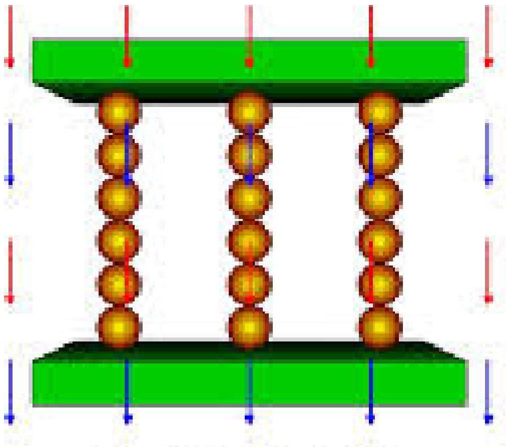


Fig 2. Without Magnetic Field Fig 3. With Magnetic Field

1.5.1 APPLICATION AREA OF MAGNETORHEOLOGICAL ELASTOMERS

- Mechanical Engineering.
- Military sector.
- Optics.
- Automobile industry.
- Aerospace field.

1.5.2 LIMITATIONS OF MAGNETORHEOLOGICAL ELASTOMERS

- Due to the presence of iron particles, the MRE's fluid become heavy.
- The high quality fluids are expensive in nature.
- After a period, the fluid become thicker in size.
- The settling down of iron particles might be a problem in some cases.

2.1 LITERATURE REVIEW:

2.1.1 SANDWICH BEAM

Di Taranto [11] formulated a procedure to calculate natural frequencies, loss factors for a finite length of sandwich beam. The damped natural frequencies and logarithmic decrement for the fundamental mode of vibration of a simply supported sandwich beam was calculated by Chatterjee and Baungarten [9]. They also did experiments to verify their theoretical results, which showed good agreement with their theoretical part. Asnani and Nakra [5] found the effect on the number of layers and thickness ratio on the system loss factors for a simply supported multilayered beam. Vaswani et al. [6] formulated the equations of motion for a multilayer curved sandwich beam which is been subjected to harmonic excitation. Lall et al. [20] analyzed the partially covered sandwich beams using three different methods and found that method by Marcus [21] estimates modal loss factor only, whereas Rayleigh-Ritz and classical search method give both loss factor and resonant frequencies. Dewa et al. [10] studied the damping effectiveness of partially covered sandwich beams. They found that partially covered beams have better damping capacity than fully covered beams. He also proved the validation by experimental results. Bhimaraddi [8] solved both the resonant frequencies and loss factors for a simply supported beam with constrained layer damping using a model, which accounted for the continuity of displacements and the transverse shear stresses across the interfaces of the layers.

Ungar [28] derived general expressions in terms of the properties of the constituting materials to find the loss factors of uniform linear composites. Rao [23] investigated the influence of pretwist on the resonant frequency and loss factor for a symmetrical pretwisted simply supported sandwich beam and prove that pretwisting reduces the loss factor and very soft thick cored beams is especially sensitive to even small changes of pretwist. Rao [2] in another work plotted graphs and gave equations to estimate frequencies and loss factors for sandwich beam under various boundary conditions. Fasana and Marchesiello [12] used Rayleigh-Ritz method to find out the mode shapes, frequencies and loss factors for sandwich beams. They choose polynomials in the sandwich beam, which satisfied the geometric boundary conditions as admissible function.

Kerwin [19] carried out a quantitative analysis of the damping effectiveness of a constrained viscoelastic layer and he found an expression to calculate the loss factor. Rao and Stühler [24] analyzed the damping effectiveness of tapered sandwich beam with simply supported and clamped free end conditions. Rubayi and Charoenee [26] studied both theoretical and experimental to obtain the natural frequencies of cantilever sandwich beams subjected to gravity force only. Johnson [16-17] analyzed the frequencies and loss factors for beams and plates with constrained viscoelastic layer by finite element method. Imaino and Harrison [15] adopted modal strain energy method and finite element technique to investigate damping of the first and second bending resonance of a sandwich beam with constrained damping layer. Jones et al. [18] investigated both theoretically and experimentally the damping capacity of a sandwich beam with viscoelastic core. Rao [25] investigated the free vibration of a short sandwich beam considering the higher order effects such as inertial, extension and shear of all the constrained layers. He found that if these parameters are neglected for short sandwich beam there is an error as high as 45% in estimation of the loss factor and frequencies. He and Rao [13] carried out a parameter study of the coupled flexural and longitudinal vibration of a curved sandwich beam. The study indicates the effect of curvature, core thickness and adhesive shear modulus on the system loss factors and resonant frequencies. He and Rao [14] in another work studied the vibration of multispan beams with arbitrary boundary condition. Effects of parameter like location of intermediate supports and adhesive thickness on the resonant frequencies and loss factors were investigated.

Asnani and Nakra [4] did analysis on multilayer simply supported sandwich beams and estimated loss factors and displacement response effectiveness for beams of different number of layers. Sakiyama et al. [27] developed a method analytically for free vibration analysis of a three layer continuous sandwich beam and investigated the effect of shear parameter and core thickness on the resonant frequencies and loss factors. Banerjee [3] investigated the free vibration of a three layer sandwich beam using dynamic stiffness matrix method. He calculated the natural frequencies and mode shapes. Nakra and Grootenhuis [22] analyzed theoretically as well as experimentally, the characteristics of asymmetric dual core sandwich beams in vibration. They ignored the rotary and longitudinal inertia terms in their analysis. Later Rao [25] included both these effects in his analysis. Rao and He [7] analyzed the sandwich beam in the dynamic state and of laminated composite simply supported beams with multiple damping layers. They found out the solution for the resonance frequencies and modal loss factors using energy and Ritz method. Rao and his colleagues studied the variation of dynamic stiffness and modal loss factor of the system with structural parameters, operating temperature zone, and damping material characteristics. Finally experiment was conducted and compared with theory. A good agreement was achieved between predicted and measured natural frequencies. Nakra et al. [35] studied the effect of curvature, lack of symmetry, core thickness and modal number on the resonant frequency parameter and associated loss factor. Lall et al. [36] with the help of Rayleigh-Ritz Method and classical Euler Theorem found the core loss factor for a partially covered sandwich beam.

2.1.2. SANDWICH BEAM WITH MAGNETORHELOGICAL ELASTOMER

Chen et al. [29] developed natural rubber based MRE's to develop the mechanical properties by considering different percentage of iron particles and reported that with the increase in iron particles the shear modulus of the MRE get increased. Nayak et al. [30] found the effects of magnetic fields on the dynamic characteristics of the sandwich beam. The work would be useful in the passive and active vibration reduction application. Nayak et al. [31] suggest the stability of the MRE embedded sandwich beam can be improved by using magnetic field. Davis [32] did the point dipole method to calculate the shear modulus of the sandwich beam using the magnetic field applied in the beam. Rajamohan et al. [33] results suggests that the natural frequencies along with the transverse displacement of the partially treated MR beams are not only influenced by the magnetic field, but also by the location and length of the fluid pocket. Ulicny et al. [34] the iron subjected due to oxidation, there is a gradual loss of fan clutch torque capacity. Sun et al. [37] studied the relationship between the magnetic field and the complex shear modulus of the MR fluids. They studied the vibration minimization capabilities of the MR beam at different magnetic fields. Yalcintas et al. [38] studied and presented a detailed analysis of vibration control capabilities of adaptive structures based on MR and ER materials, and compared their vibration minimization rates, time responses and energy consumption rates.

2.2 BLUEPRINT OF THE PRESENT WORK:

The present work deals with the buckling and free vibration analysis of a beam with partially covering constraining layer with magnetorheological fluid core. For the analysis a fixed free cantilever was been used. The cantilever beam was partially covered and the equations of motion were derived using Finite Element method in conjunction with Hamilton’s principle. The displacement found out were used to find the global mass and stiffness matrix which was eventually used to find out the natural frequency. The first three mode of frequency was found out theoretically using MATLAB code. The patch at a distance of 3L/4 shows the maximum frequency in the three cases. The results found out for 3L/4 patch were used to look into the effect of magnetic field on the frequency. It was seen that the frequency increases with the increase in magnetic field. Also the buckling capacity of the beam gets increased when the patch is attached in the L/2 length. The experimental verification was done to verify the increase in frequency with increase in magnetic field.

3.1 FORMULATION OF THE PROBLEM:

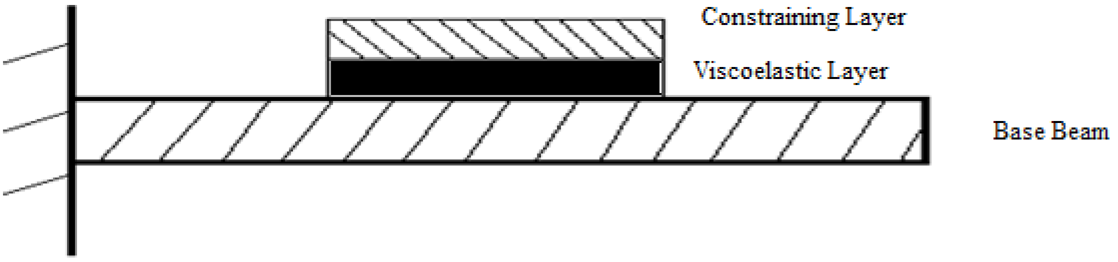


Fig 4. Cantilevered layered Sandwich Beam

Figure (4) shows a three layered cantilevered symmetric sandwich beam of length L. The finite element model is been developed on the following assumptions:

- The rotary inertia and shear deformations in the constraining layers are assumed to be negligible.
- Linear theories of elasticity and viscoelasticity is been used.
- In the layers no slip occurs and there is perfect continuity at the interfaces.

3.2 Element matrices of the three layered Beam

As shown in figure the element model shown here consists of two nodes and each node has four degrees of freedom. Nodal displacements are given by

$$\{\Delta^e\} = \{u_{1i}\ u_{3i}\ w_{1i}\ w_{3i}\ \theta_{1i}\ \theta_{3i}\ u_{1j}\ u_{3j}\ w_{1j}\ w_{3j}\ \theta_{1j}\ \theta_{3j}\}^T \quad (1)$$

where i and j are the elemental nodal numbers. The axial displacement of the constraining layer, the transverse displacement and the rotational angle, are expressed in terms of nodal displacements and finite element shape functions.

$u_1 = [\ Nu_1\ ]\ \{\Delta^e\}$, $u_3 = [\ Nu_3\ ]\ \{\Delta^e\}$, $w_1 = [\ N_{w1}\ ]\ \{\Delta^e\}$, $w_3 = [\ N_{w3}\ ]\ \{\Delta^e\}$,

$\theta_1 = [N_w]'\{\Delta^e\}$, $\theta_3 = [N_{w3}]'\{\Delta^e\}$ (2)

where the prime denotes differentiation with respect to axial coordinate x and the shape functions are shown below:

$[Nu_1] = [1-\xi\ 0\ 0\ 0\ 0\ 0\ \xi\ 0\ 0\ 0\ 0\ 0]$

$[Nu_3] = [0\ 1-\xi\ 0\ 0\ 0\ 0\ 0\ \xi\ 0\ 0\ 0\ 0]$

$[N_{w1}] = [\ 0\ \ 0\ \ (1-3\xi^2+2\xi^3)\ \ 0\ (\xi-2\xi^2+\xi^3)L_e\ \ 0\ \ 0\ \ 0\ 3\xi^2-2\xi^3\ \ 0\ \ (-\xi^2+\xi^3)\ L_e\ \ 0\ ]$

$[N_{w3}] = [\ 0\ \ 0\ \ 0\ (1-3\xi^2+2\xi^3)\ \ 0\ (\xi-2\xi^2+\xi^3)L_e\ \ 0\ \ 0\ \ 0\ 3\xi^2-2\xi^3\ \ 0\ \ (-\xi^2+\xi^3)\ L_e\ ]$

where $\xi = x / L$ and L is the length of the element.

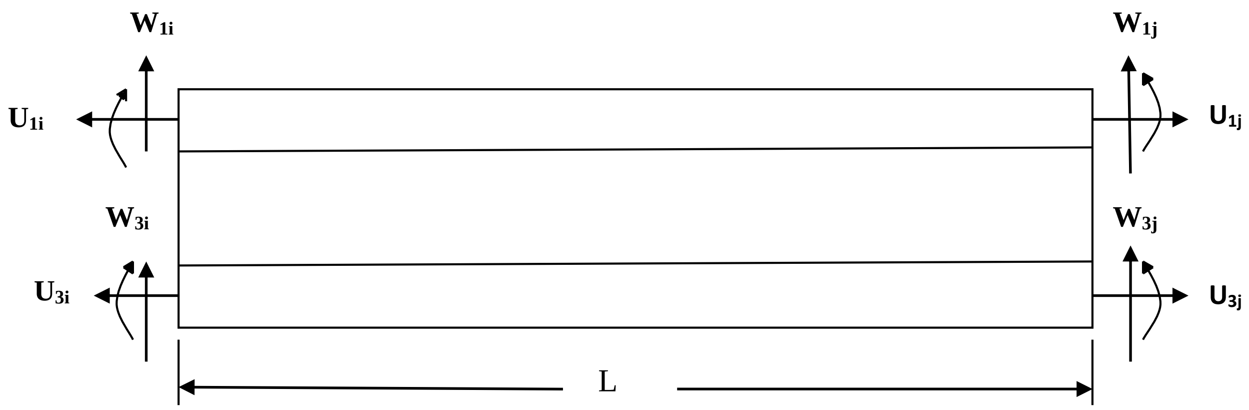


Fig.5 Finite Beam Element for a three layered Sandwich Beam

The element potential energy ($U^{(e)}$) of the beam is equal to sum of potential energy of the constraining layers and viscoelastic layers.

$U^{(e)} = U_c^{(e)} + U_v^{(e)}$ (3)

3.2.1 Element Matrices of constraining layers:

The potential energy of the constrained layers is been written as:

$$U_k^{(e)} = \frac{1}{2}\int_0^l E_k I_k \left[\frac{d^2 w_k}{dx^2}\right]^2 dx + \frac{1}{2}\int_0^l E_k A_k \left[\frac{du_k}{dx}\right]^2 dx \qquad k = 1,3 \tag{4}$$

where E, A and I are the Young's modulus of the beam, cross-sectional area and moment of inertia respectively. The notations 1 and 3 represents the upper and lower constraining layer, respectively.

The kinetic energy of the constraining layers is been written as

$$T_k^{(e)} = \frac{1}{2}\int_0^l \rho_k A_k \left[\frac{dw_k}{dt}\right]^2 dx + \frac{1}{2}\int_0^l \rho_k A_k \left[\frac{du_k}{dt}\right]^2 dx \qquad k = 1,3 \tag{5}$$

where ρ is the mass density.

By substituting Eq. (2) in to Eq. (4) and Eq. (5), the elemental potential energy and the kinetic energy of the constraining layers can be written as

$$U_k^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}\left(\left[K_{ku}^{(e)}\right] + \left[K_{kw}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1,3 \tag{6}$$

and

$$T_k^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}\left(\left[M_{ku}^{(e)}\right] + \left[M_{kw}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1,3 \tag{7}$$

where,

$$\left[K_{ku}^{(e)}\right] = \left[K_{1u}^{(e)}\right] + \left[K_{3u}^{(e)}\right] = E_1 A_1 \int_0^l [N_1]^T [N_1] dx + E_3 A_3 \int_0^l [N_3]^T [N_3] dx$$

$$\left[K_{kw}^{(e)}\right] = \left[K_{1w}^{(e)}\right] + \left[K_{3w}^{(e)}\right] = E_1 I_1 \int_0^l [N_w]''^T [N_w]'' dx + E_3 I_3 \int_0^l [N_w]''^T [N_w]'' dx \tag{8}$$

$$\left[M_{ku}^{(e)}\right] = \left[M_{1u}^{(e)}\right] + \left[M_{3u}^{(e)}\right] = \rho_1 A_1 \int_0^l [N_1]^T [N_1] dx + \rho_3 A_3 \int_0^l [N_3]^T [N_3] dx$$

$$\left[M_{kw}^{(e)}\right] = \left[M_{1w}^{(e)}\right] + \left[M_{3w}^{(e)}\right] = \rho_1 A_1 \int_0^l [N_w]^T [N_1] dx + \rho_3 A_3 \int_0^l [N_w]^T [N_3] dx$$

and the dot denotes the differentiation with respect to time t.

3.2.2 Element Matrices of viscoelastic layer:

The potential energy of Viscoelastic layer due to shear deformation is written as

$$U_v^{(e)} = \frac{1}{2}\int_0^l G_v A_v \gamma_v^2 dx + \frac{1}{2}\int_0^l E_v A_v \varepsilon_{vL}^2 dx + \frac{1}{2}\int_0^l G_v A_v \varepsilon_{vT}^2 dx \quad (9)$$

where A_v is the cross-sectional area and G_v is the complex shear modulus of viscoelastic layer. $G_v = G_v^*[1+i\,(\eta_c)]$. G_v^* is the in-phase shear modulus of the viscoelastic material layer and η_c is the associated core loss factor and $i=\sqrt{-1}$.

For sealing of MR fluid a silicone gel was applied at the edges having uniform thickness to hold the MR fluid within the beam. The middle layer of the sandwich beam contains the silicone seal and the MR fluid, however, it is considered as homogenous material layer having equivalent shear modulus and can be represented by moduli and width of two material such as,

$$\overline{G} = G_r\left(\frac{b_r}{b}\right) + G^*(1 - \frac{b_r}{b})$$

where, *Gr* and *G** are the shear modulus of the rubber and MR fluid respectively. $\overline{G}$ is the equivalent shear modulus of the homogeneous layer, b_r and b are the widths of the rubber and entire beam respectively,. In the pre yield region, the MR material exhibits viscoelastic behavior, which can be described in terms of the complex modulus G^* and given by,

$$G^* = G' + iG''$$

where, G' is storing modulus of the MR fluid, which represent during a deformation cycle average energy stored per unit volume of the material, and G'' is the loss modulus, it represent the energy dissipated per unit volume of the material over a cycle.

The simulation is performed for a sandwich beam with MR core with a dimensions of elastic layer is 300mm×30mm×1mm with an identical thickness of MR fluid layers. The various properties of material are,

$E_1=E_3=68GPa; \rho_1 = \rho_3 = 2700 kg/m^3; \rho_2 = 3500 kg/m^3; \rho_r = 1233 kg/m^3;$

There exist a relation between a shear modulus and applied magnetic field and is given by,

$$G'(B) = -3.3691B^2 + 4.9975 \times 10^3\ B + 0.893 \times 10^6$$

$$G''(B) = -0.9B^2 + 0.8124 \times 10^3\ B + 0.1855 \times 10^6$$

The shear strain γ_V, longitudinal strain (ε_{vL}) and transverse shear strain (ε_{vT}) due to thickness deformation in the viscoelastic layer from kinematic relationship between the constraining layers shown as follows:

$$\gamma_v = \frac{u_1 - u_3}{2} + \frac{(t_1 + t_2)}{2t_2}\frac{\partial w_1}{\partial x} + \frac{(t_2 + t_3)}{2t_2}\frac{\partial w_3}{\partial x}$$

$$\varepsilon_{vL} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x}\right) + \frac{t_1}{4}\left(\frac{\partial w_1}{\partial x}\right) - \frac{t_3}{4}\left(\frac{\partial w_3}{\partial x}\right) \tag{10}$$

$$\varepsilon_{vT} = \left(\frac{w_1 - w_3}{t_2}\right)$$

Substituting Eq. (2) in to Eq. (10) γ_v, ε_{vL}, ε_{vT} and u_v can be expressed in terms of nodal displacements and element shape functions:

$$\gamma_v = [N_\gamma]\{\Delta^{(e)}\}$$

$$\varepsilon_{vL} = [N_L]\{\Delta^{(e)}\} \tag{11}$$

$$\varepsilon_{vT} = [N_T]\{\Delta^{(e)}\}$$

where

$$[N_\gamma] = \frac{([N_{u1}] - [N_{u1}])}{t_2} + \frac{(t_1 + t_2)}{2t_2}[N_{w1}] - \frac{(t_3 + t_2)}{2t_2}[N_{w3}]$$

$$[N_L] = \frac{1}{2}([N_{u1}]' + [N_{u3}]') + \frac{t_1}{4}[N_{w1}]'' - \frac{t_3}{4}[N_{w3}]'' \tag{12}$$

$$[N_T] = \frac{1}{t_2}([N_{w1}] - [N_{w3}])$$

Substituting eq. (11) in to eq. (7) the potential energy of the viscoelastic material layer is shown as

$$U_v^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}^T\left(\left[K_{v\gamma}^{(e)}\right] + \left[K_{vL}^{(e)}\right] + \left[K_{vT}^{(e)}\right]\right)\{\Delta^{(e)}\} \tag{13}$$

where,

$$\left[K_{v\gamma}^{(e)}\right] = G_v A_v \int_0^l [N_\gamma]^T [N_\gamma] dx$$

$$\left[K_{vL}^{(e)}\right] = E_v A_v \int_0^l [N_L]^T [N_L] dx \tag{14}$$

$$\left[K_{vT}^{(e)}\right] = E_v A_v \int_0^l [N_T]^T [N_T] dx$$

The kinetic energy of the viscoelastic layer is shown as below

$$T_v^{(e)} = \frac{1}{2}\int_0^l \rho_v A_v \left\{\left[\frac{\partial w_v}{\partial t}\right]^2 + \left[\frac{\partial u_v}{\partial t}\right]^2\right\} dx \tag{15}$$

where A_v is the cross-sectional area and ρ_v is the mass density of the viscoelastic layer. The axial and lateral displacement u_v and w_v of the viscoelastic layer were derived from kinematic relationships between the constraining layers is expressed as follows:

$$u_v = \frac{u_3 + u_1}{2} + \frac{t_1}{4}\frac{\partial w_1}{\partial x} - \frac{t_3}{4}\frac{\partial w_3}{\partial x} \tag{16}$$

$$w_v = \frac{w_3 + w_1}{2}$$

Substituting eq. (2) in to eq. (16) u_v and w_v can be expressed in terms of nodal displacements and elemental shape functions:

$$u_v = [N_{vL}]\{\Delta^{(e)}\} \tag{17}$$

$$w_v = [N_{vT}]\{\Delta^{(e)}\}$$

where,

$$\left.\begin{aligned} [N_{vL}] &= \frac{1}{2}([N_{u3}] + [N_{u1}]) + \frac{t_1}{4}[N_{w1}] - \frac{t_3}{4}[N_{w3}] \\ [N_{vT}] &= \frac{1}{2}([N_{w3}] + [N_{w1}]) \end{aligned}\right\} \tag{18}$$

Substituting eq. (2) in to eq. (17) and (15), the kinetic energy of viscoelastic material layers is shown as

$$T_v^{(e)} = \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M_{vL}^{(e)}\right] + \left[M_{vT}^{(e)}\right]\right)\{\dot{\Delta}^{(e)}\} \tag{19}$$

where,

$$\left[M_{vL}^{(e)}\right] = \rho_v A_v \int_0^l [N_{vL}]^T [N_{vL}] dx \tag{20}$$

$$\left[M_{vT}^{(e)}\right] = \rho_v A_v \int_0^l [N_{vT}]^T [N_{vT}] dx$$

The total potential energy of the element is shown below:

$$U^{(e)} = \sum_{k=1,3} \frac{1}{2}\{\Delta^{(e)}\}^T \left(\left[K^{(e)}_{(k)u}\right] + \left[K^{(e)}_{(k)w}\right]\right)\{\Delta^{(e)}\} + \frac{1}{2}\{\Delta^{(e)}\}^T\left(\left[K^{(e)}_{v\gamma}\right] + \left[K^{(e)}_{vL}\right] + \left[K^{(e)}_{vT}\right]\right)\{\Delta^{(e)}\}$$

$$U^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}^T\left[K^{(e)}\right]\{\Delta^{(e)}\} \tag{21}$$

where,

$$\left[K^{(e)}\right] = \sum_{k=1,2} \left(\left[K^{(e)}_{(2k-1)u}\right] + \left[K^{(e)}_{(2k-1)w}\right]\right) + \left[K^{(e)}_{v\gamma}\right] + \left[K^{(e)}_{vL}\right] + \left[K^{(e)}_{vT}\right] \tag{22}$$

$\left[K^{(e)}\right]$ is the element stiffness matrix.

The total kinetic energy of the element is shown below:

$$T^{(e)} = \sum_{k=1,3} \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M^{(e)}_{(k)u}\right] + \left[M^{(e)}_{(k)w}\right]\right)\{\dot{\Delta}^{(e)}\} + \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M^{(e)}_{vL}\right] + \left[M^{(e)}_{vT}\right]\right)\{\dot{\Delta}^{(e)}\}$$

$$T^{(e)} = \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T\left[M^{(e)}\right]\{\dot{\Delta}^{(e)}\} \tag{23}$$

where,

$$\left[M^{(e)}\right] = \sum_{k=1,2} \left(\left[M^{(e)}_{(2k-1)u}\right] + \left[M^{(e)}_{(2k-1)w}\right]\right) + \left(\left[M^{(e)}_{vL}\right] + \left[M^{(e)}_{vT}\right]\right) \tag{24}$$

$\left[M^{(e)}\right]$ is the elemental mass matrix.

3.2.3 Element geometric stiffness matrix:

The elemental work done by axial force *P* is written as:

$$W_p^{(e)} = \frac{1}{2}\int_0^l P\left[\left(\frac{\partial w_1}{\partial x}\right)^2 + \left(\frac{\partial w_3}{\partial x}\right)^2\right]dx \tag{25}$$

The work done by the axial load can be rewritten as:

$$W_p^{(e)} = \frac{1}{2}\left\{\Delta^{(e)}\right\}^T \; K_g^{(e)}\left\{\Delta^{(e)}\right\} \tag{26}$$

where

$$[K_g^{(e)}]=\int_0^l [N_{w1}]'^T [N_{w1}]' + [N_{w3}]'^T [N_{w3}]' \quad dx \tag{27}$$

is the elemental geometric stiffness matrix.

3.3 Element matrices of the single layered Beam:

From figure (4) we can clearly see that the sandwich beam is not three layered for the whole length. There is patch wise three layered beam, therefore for the single layered we have to consider the bottom beam without any effect of the viscoelastic layer. Nodal displacement are given by:

$$\{\Delta^e\} = \{u_{11} \;\; w_{11} \;\; \theta_{11} \;\; u_{12} \;\; w_{12} \;\; \theta_{12}\}^T \tag{28}$$

where 1 and 2 are the elemental nodal numbers. The axial displacement of the constraining layer, the transverse displacement and the rotational angle, are expressed in terms of nodal displacements and finite element shape functions.

$$u_1=[N_{u1}]\{\Delta^e\}, w_1=[N_{w1}]\{\Delta^e\}, \theta_1= [N_{w1}]'\{\Delta^e\}, \tag{29}$$

where the prime denotes differentiation with respect to axial coordinate x and the shape functions are shown below:

$[N_{u1}] = [1- \xi \;\; 0 \;\; 0 \;\; \xi \;\; 0 \;\; 0]$

$[N_{w1}] = [0 \quad (1-3\xi^2+2\xi^3) \quad (\xi-2\xi^2+\xi^3)L_e \quad 0 \quad 3\xi^2-2\xi^3 \quad (-\xi^2+\xi^3) L_e]$

where $\xi = x / L$ and L is the length of the element.

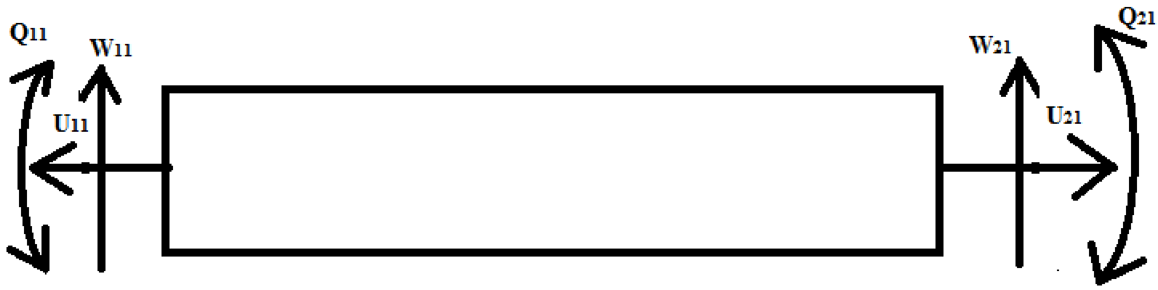


Fig. 6 Finite Beam Element for a single Layered Sandwich Beam

The element potential energy ($U^{(e)}$) of the beam is equal to sum of potential energy of the constraining layers.

$$U^{(e)} = U_c^{(e)} \tag{30}$$

3.3.1 Element Matrices of constraining layers:

The potential energy of the constrained layers is been written as:

$$U_k^{(e)} = \frac{1}{2}\int_0^l E_k I_k \left[\frac{d^2 w_k}{dx^2}\right]^2 dx + \frac{1}{2}\int_0^l E_k A_k \left[\frac{du_k}{dx}\right]^2 dx \qquad k = 1 \tag{31}$$

where E, A and I are the Young's modulus of the beam, cross-sectional area and moment of inertia respectively. The notations 1 represents the lower constraining layer, respectively.

The kinetic energy of the constraining layers is been written as

$$T_k^{(e)} = \frac{1}{2}\int_0^l \rho_k A_k \left[\frac{dw_k}{dt}\right]^2 dx + \frac{1}{2}\int_0^l \rho_k A_k \left[\frac{du_k}{dt}\right]^2 dx \qquad k = 1 \tag{32}$$

where ρ is the mass density.

By substituting Eq. (29) in to Eq. (31) and Eq. (32), the elemental potential energy and the kinetic energy of the constraining layers can be written as

$$U_k^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}\left(\left[K_{ku}^{(e)}\right] + \left[K_{kw}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1 \tag{33}$$

and

$$T_k^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}\left(\left[M_{ku}^{(e)}\right] + \left[M_{kw}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1 \tag{34}$$

where,

$$\left[K_{ku}^{(e)}\right] = \left[K_{1u}^{(e)}\right] = E_1 A_1 \int_0^l [N_1]^T\,[N_1]dx$$

$$\left[K_{kw}^{(e)}\right] = \left[K_{1w}^{(e)}\right] = E_1 I_1 \int_0^l [N_w]''^T\,[N_w]''dx \tag{35}$$

$$\left[M_{ku}^{(e)}\right] = \left[M_{1u}^{(e)}\right] = \rho_1 A_1 \int_0^l [N_1]^T\,[N_1]dx$$

$$\left[M_{kw}^{(e)}\right] = \left[M_{1w}^{(e)}\right] = \rho_1 A_1 \int_0^l [N_w]^T\,[N_1]dx \tag{36}$$

and the dot denotes the differentiation with respect to time t.

The total potential energy of the element is shown below:

$$U^{(e)} = \sum_{k=1} \frac{1}{2}\{\Delta^{(e)}\}^T\left(\left[K_{(k)u}^{(e)}\right] + \left[K_{(k)w}^{(e)}\right]\right)\{\Delta^{(e)}\}$$

$$U^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}^T\left[K^{(e)}\right]\{\Delta^{(e)}\} \tag{37}$$

where,

$$\left[K^{(e)}\right] = \sum_{k=1}\left(\left[K_{(2k-1)u}^{(e)}\right] + \left[K_{(2k-1)w}^{(e)}\right]\right) \tag{38}$$

$\left[K^{(e)}\right]$ is the element stiffness matrix.

The total kinetic energy of the element is shown below:

$$T^{(e)} = \sum_{k=1} \frac{1}{2} \left\{\dot{\Delta}^{(e)}\right\}^{T} \left(\left[M_{(2k-1)u}^{(e)}\right] + \left[M_{(2k-1)w}^{(e)}\right]\right) \left\{\dot{\Delta}^{(e)}\right\}$$

$$T^{(e)} = \frac{1}{2} \left\{\dot{\Delta}^{(e)}\right\}^{T} \left[M^{(e)}\right] \left\{\dot{\Delta}^{(e)}\right\} \tag{39}$$

where,

$$\left[M^{(e)}\right] = \sum_{k=1} \left(\left[M_{(2k-1)u}^{(e)}\right] + \left[M_{(2k-1)w}^{(e)}\right]\right) \tag{40}$$

$\left[M^{(e)}\right]$ is the elemental mass matrix.

The elemental work done by axial force *P* is written as

$$W_p^{(e)} = \frac{1}{2} \int_0^l P \left[\left(\frac{\partial w_1}{\partial x}\right)^2\right] dx \tag{41}$$

Work done by the axial load can be rewritten as

$$W_p^{(e)} = \frac{1}{2} \left\{\Delta^{(e)}\right\}^T \; K_g^{(e)} \left\{\Delta^{(e)}\right\} \tag{42}$$

where $[K_g^{(e)}] = \int_0^l [N_{w1}]'^T [N_{w1}]' \; dx$ (43)

is the elemental geometric stiffness matrix.

3.4 Governing Equation of motion:

The element equation of motion for a sandwich beam is obtained by using Hamilton's principle.

$$\delta \int_{t_1}^{t_2} \left(T^{(e)} - U^{(e)} + W_p{}^{(e)} \right) dt = 0 \tag{44}$$

Substituting the previous equation, the equation of motion for the sandwich beam element is obtained as follows:

$$\left[M^{(e)} \right] \left\{ \ddot{\Delta}^{(e)} \right\} + \left[K^{(e)} \right] \left\{ \Delta^{(e)} \right\} - P \left[K_g^{(e)} \right] \left\{ \Delta^{(e)} \right\} = 0 \tag{45}$$

Assembling mass, elastic stiffness and geometric stiffness matrices of individual element, the equation of motion for the beam is written as:

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} - P\left[K_g\right]\{\Delta\} = 0 \tag{46}$$

Governing equation to determine natural frequency

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} = 0$$

Governing equation to determine buckling load

$$[K]\{\Delta\} - P\left[K_g\right]\{\Delta\} = 0$$

where $\{\Delta\}$ is the global displacement matrix.

4.1 Experimental methodology:

The specimen was prepared for L/4, L/2 and 3L/4 patch to validate the results obtained by the theoretical one for magnetic field strength of 300G.

4.1.1 Specimen preparation:

The specimen was prepared in the following way:

- Aluminium sheet of (40*4*0.5) cm was made as the base beam.
- Aluminium sheet of (4*4*0.5) cm was made as the constraining layer at length of 10cm, 20cm and 30 cm.
- The top and the bottom layer was initially bonded with the help of silicone gel which was kept to be dried up for 48 hours.
- The space between the top and bottom layer was filled up with magnetorheological fluid.
- The magnetorheological fluid was prepared with the help of silica oil which was poured along with the iron particles in a definite proportion i.e. 20% of silica oil by weight.
- The magnetorheological fluid was injected into the space provided with the help of syringe.

4.1.2 Experiment procedure:

- The specimen prepared was fixed at the holder of the electro dynamic shaker machine and magnetic field strength of 300G is been applied.
- The specimen natural frequency was calculated theoretically to get a rough idea of its frequency.

- The accelerometer probe was connected to the oscilloscope at one end and to the other end to the specimen.
- The digital switching power machine was connected to the dynamic shaker machine through which the frequency can be regulated.
- The vibration was increased by the knob of frequency in the digital switching power machine.
- The amplitude is viewed in the oscilloscope.
- As the amplitude range gets increased there would be one point where there would be no further increase and there would be decrement in amplitude.
- At that particular amplitude, the frequency recorded would be the 1^{st} mode of the natural frequency.
- Similarly the next mode of frequency could be calculated.

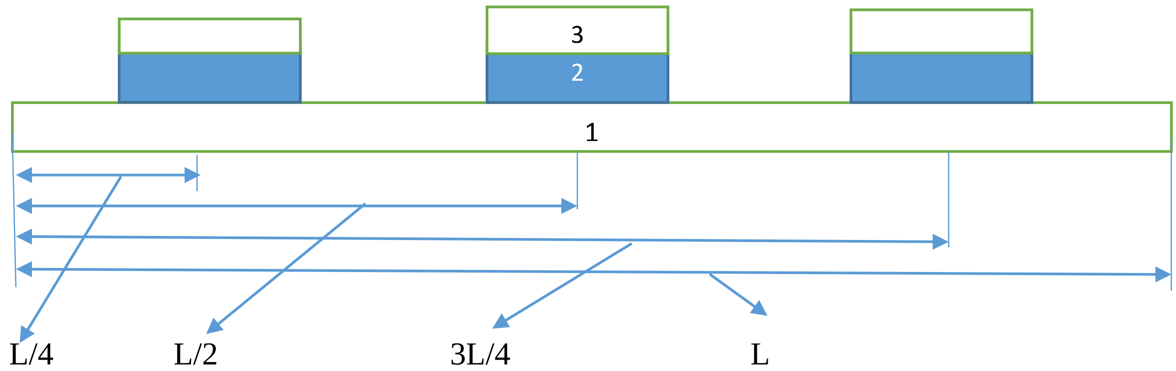


Fig.7 Schematic diagram of the specimen

In the present figure:

1. Base layer of Aluminium sheet.

2. Middle layer of MR Fluid.

3. Top layer of Aluminium sheet.

Fig. 8 Experiment Set up.

Fig.9 Experimental set up of L/2 specimen.

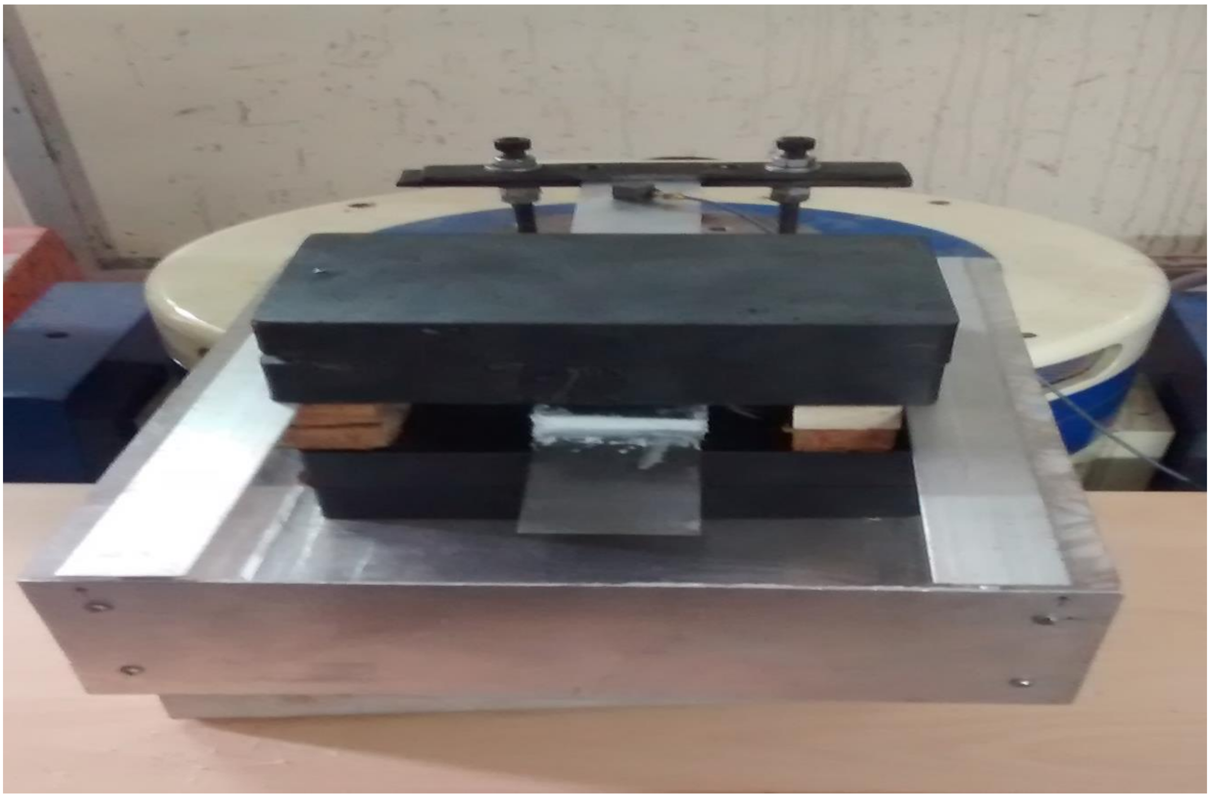


Fig. 10 Experimental set up of 3L/4 set up.

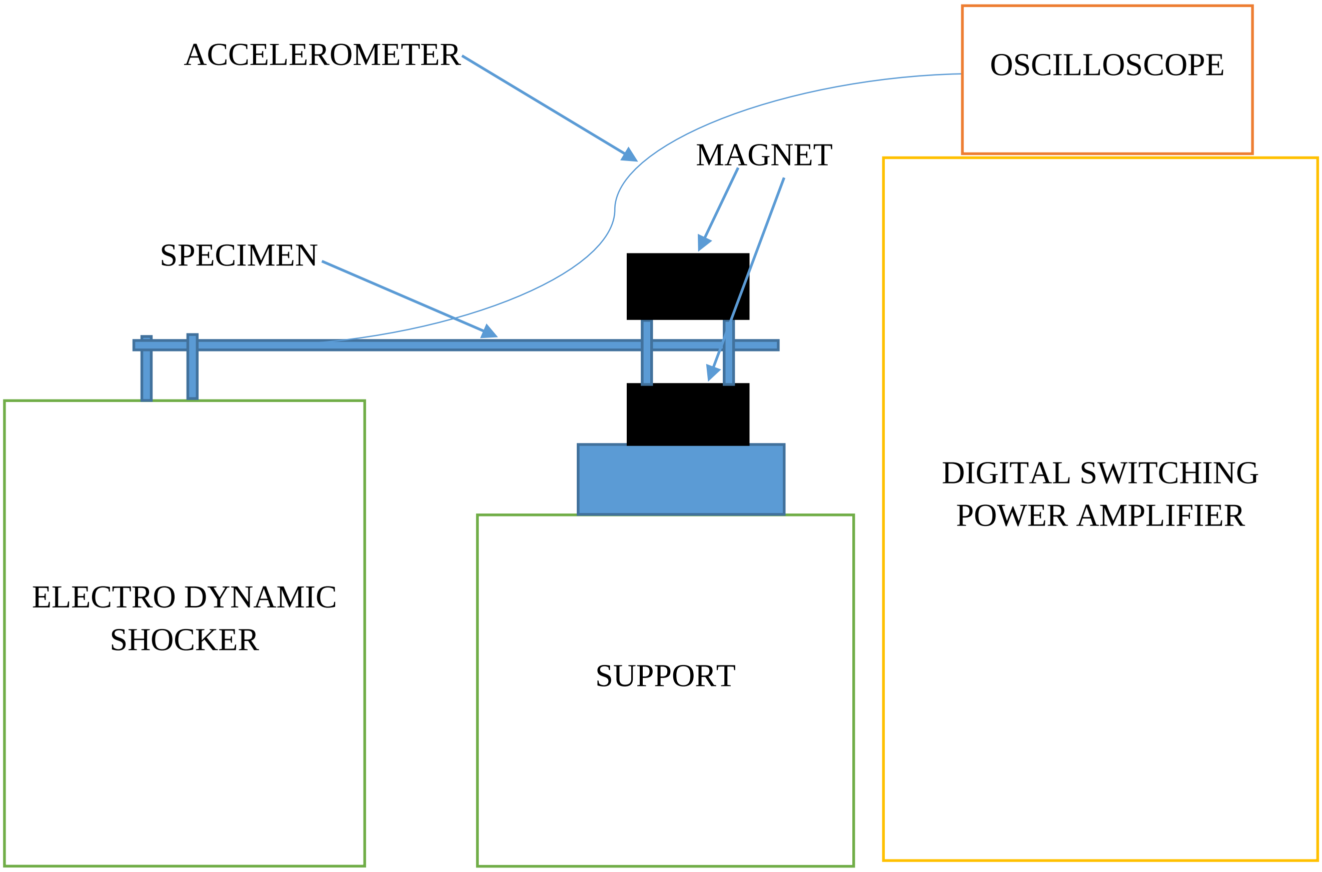


Fig. 11 Schematic diagram of the Experimental set up

5.1 RESULTS AND DISCUSSION:

The sandwich beam was discretized using Finite Elements. An eight element discretization satisfy the convergence requirement. The following graphs were obtained are shown below:

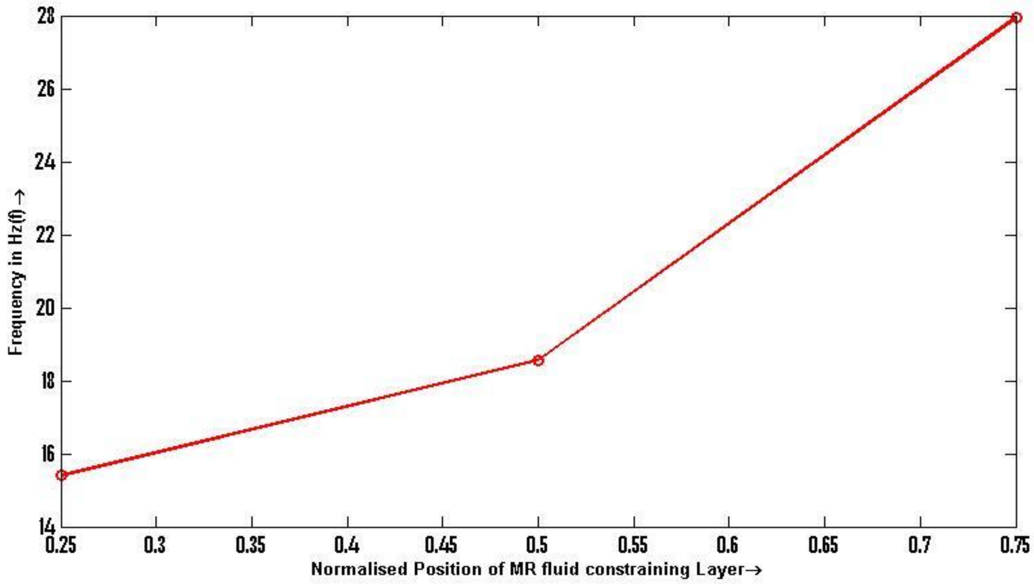


Fig. 12 shows the plot between the effects of constraining layer position on 1st mode frequency.

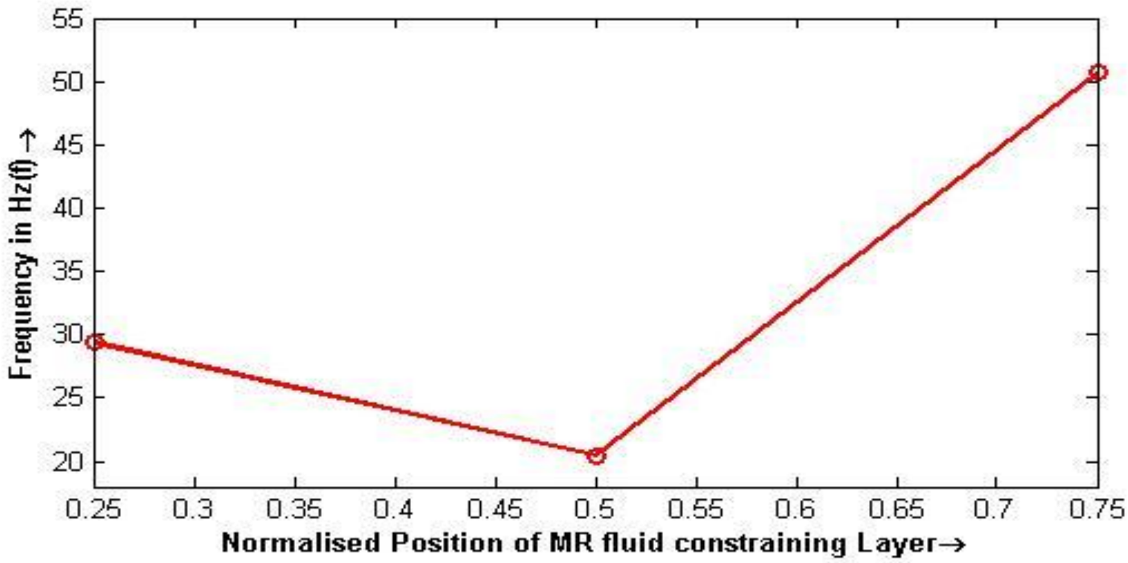


Fig.13 shows the plot between the effects of constraining layer position on 2nd mode frequency

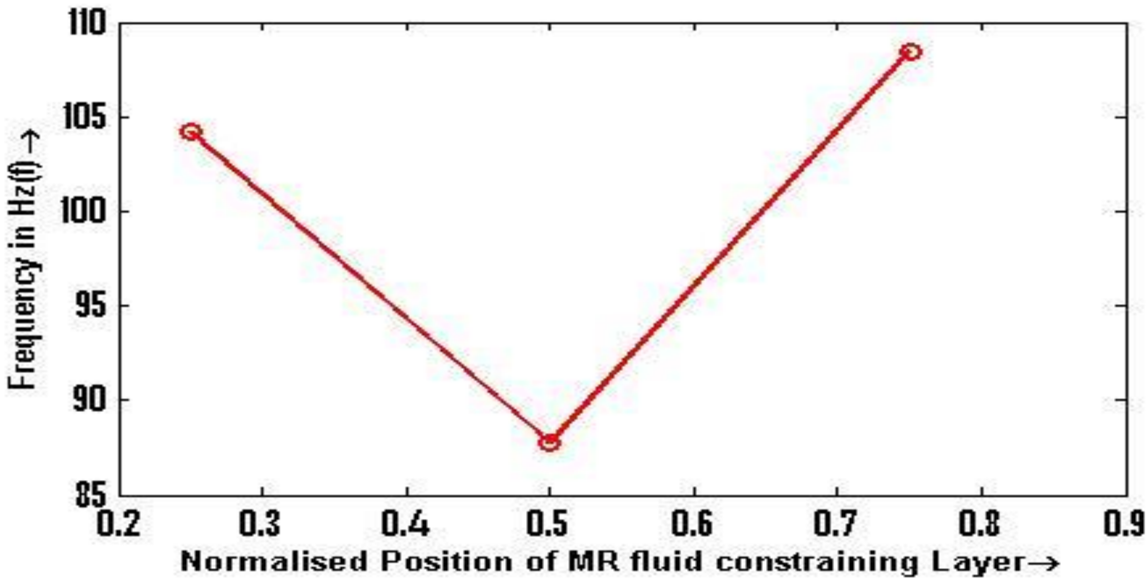


Fig.14 shows the plot between the effects of constraining layer position on 3rd mode frequency. From the above three plot it clearly indicates the highest frequency is achieved nearer to the free end for the three mode of vibration.

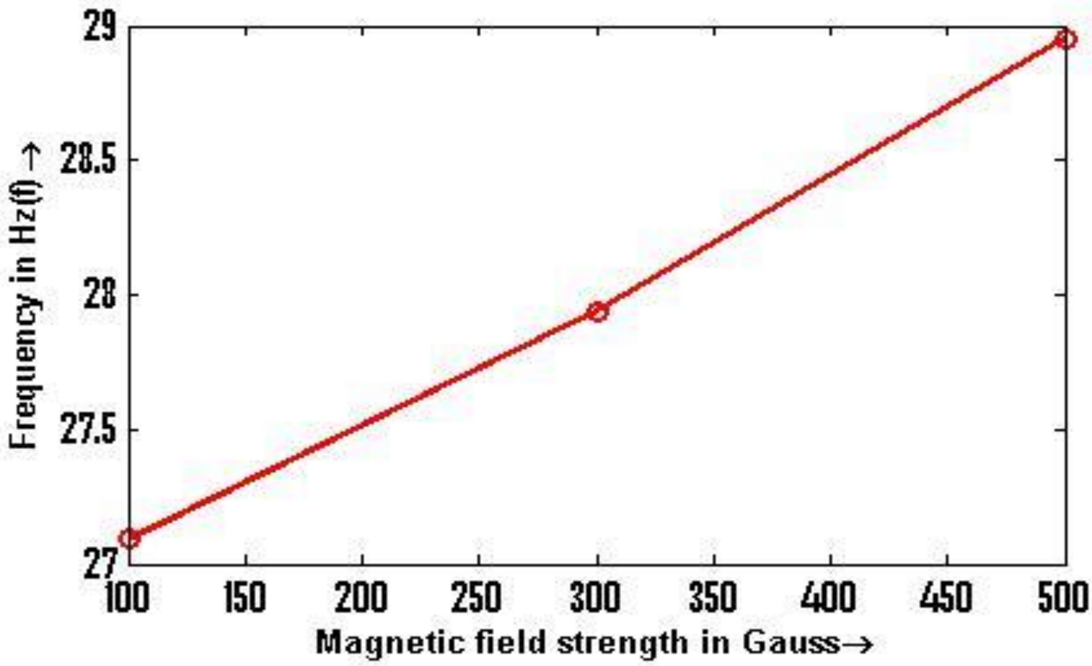


Fig.15 shows the plot between the effects of magnetic field strength on 1st mode frequency.

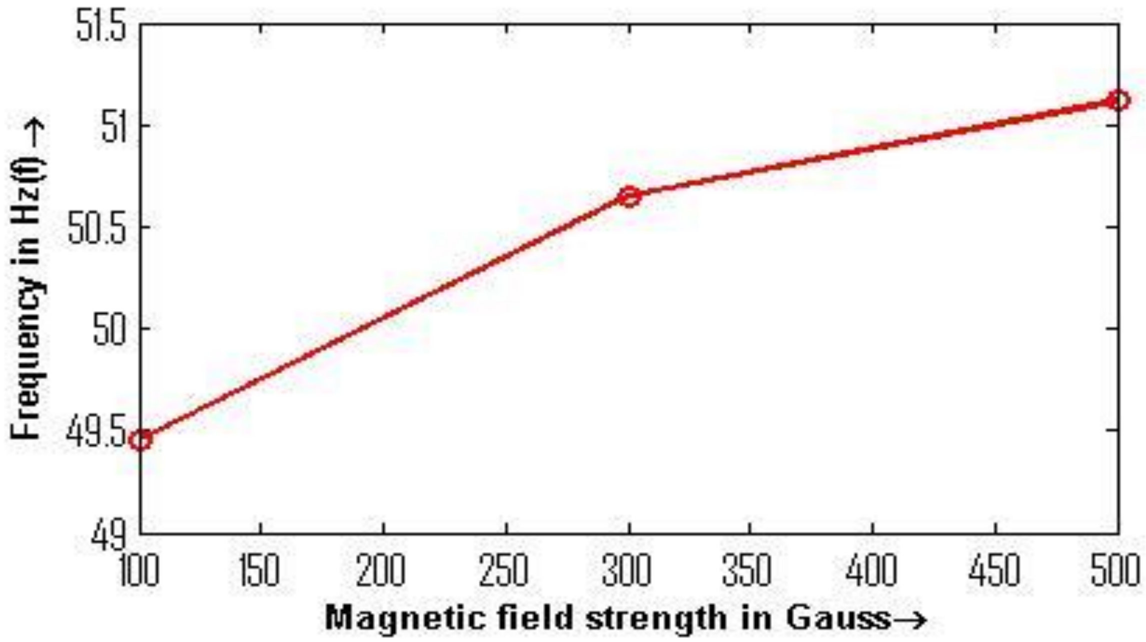


Fig.16 shows the plot between the effects of magnetic field strength on 2nd mode frequency.

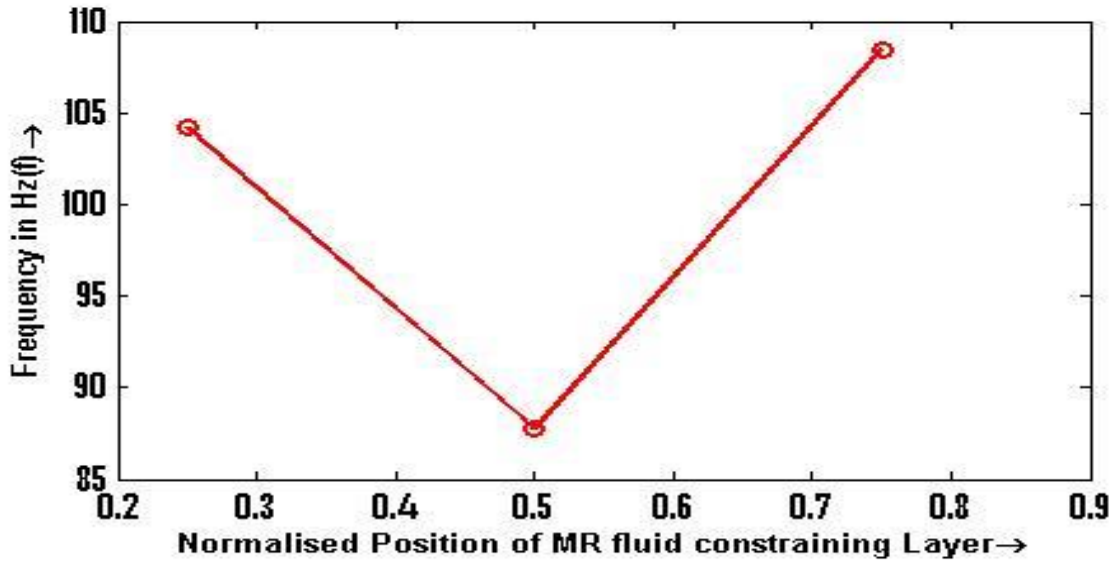


Fig.17 shows the plot between the effects of magnetic field strength on 3rd mode frequency.

From the above three plot it clearly indicates with increase in magnetic field, there is considerable increase in the first three mode of frequency of the patch in the 3/4th end of the sandwich beam with the MR fluid.

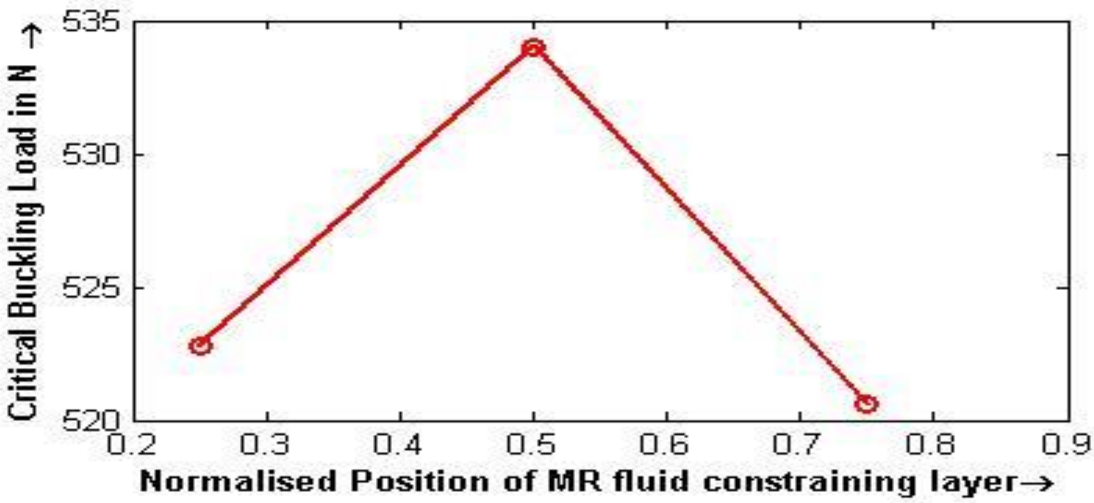


Fig.18 shows the plot between the effects of constraining layer position on fundamental buckling load. The above plot shows maximum buckling capacity is achieved by placing the constraining layer at the mid position of the beam.

Experimental results for magnetic field strength of 300G:

Constraining Layer position	**1^{st} mode frequency in Hz**		**2^{nd} mode frequency in Hz**	
	Theoretical	Practical	Theoretical	Practical
$\frac{L}{4}$	15.4	17.0	29.4	28
$\frac{L}{2}$	18.57	20.0	20.49	24
$\frac{3L}{4}$	27.94	26.0	50.65	51

Table 1: Experimental result for magnetic field strength of 300G

The discrepancy in the theoretically calculated and experimentally measured value may be due to experimental error or may be due assumptions made to derive the governing equations. However the values are very close to each other.

6.1 CONCLUSION AND FUTURE SCOPE OF WORK:

In the present case the governing equation of motion have been derived using Hamilton's principle in conjunction with finite element method.

The effect of magnetic field strength and constraining layer position on the frequency and critical buckling load of the beam has been studied. The constraining layer nearer to the free end gives highest value of frequencies in the first three mode frequency. The effect of magnetic field on the constraining layer nearer to the free end is been investigated. It has been found that with increase in magnetic field strength there is a considerable increase in the frequencies of the constraining layer nearer to the free end. Maximum buckling capacity is achieved by placing the constraining layer at the mid position of the beam.

The scope for future work can be seen as:

- Forced vibration can be used.
- Different types of MR fluids could be used.
- Dynamic stability factors can also be considered.
- Experimental results could be extended to other magnetic field strength.
- The analysis can be extended to other structural elements like plates and shells.

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